

3.15 Parametric Equations & Motion

47 points

Name Date Period

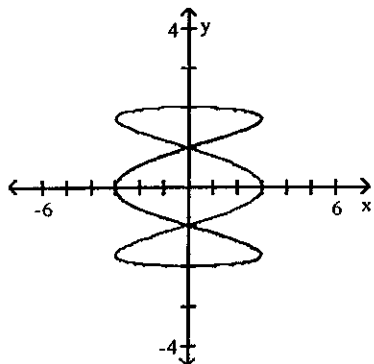
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the graph of the given parametric equations.

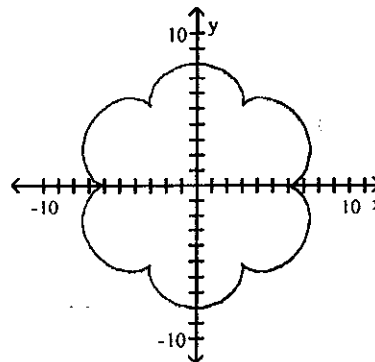
1) $x = 3 \sin^3 t$, $y = 3 \cos^3 t$

1) C

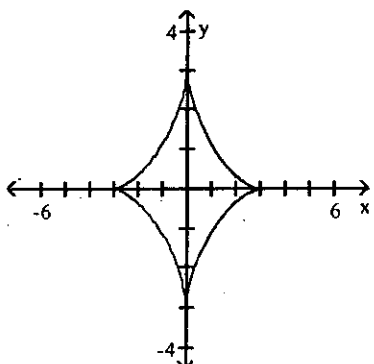
A)



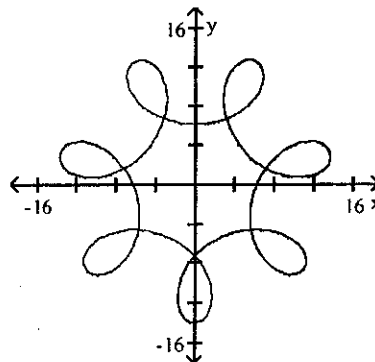
B)



C)



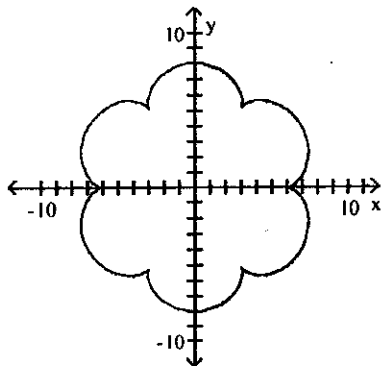
D)



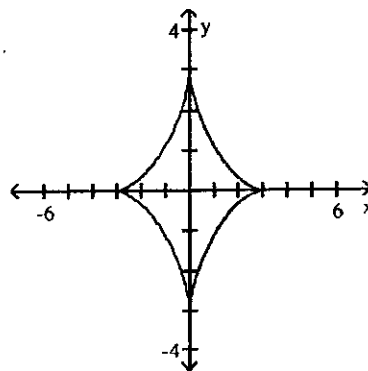
2) A

2) $x = 7 \sin t + \sin 7t, y = 7 \cos t + \cos 7t$

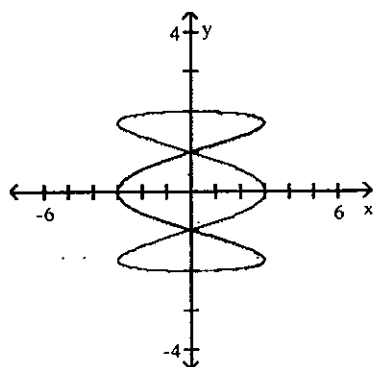
A)



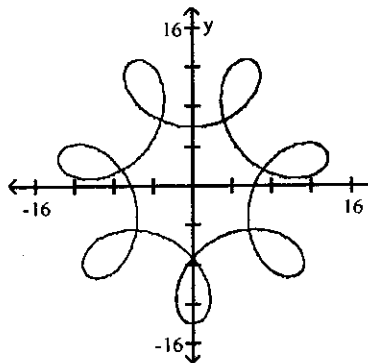
B)



C)



D)



SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Graph the pair of parametric equations.

3) $x = 2t, y = t + 4, -2 \leq t \leq 3$

$t = -2$ **End points**

$x = 2(-2) = -4$ $(-4, 2)$

$y = -2 + 4 = 2$

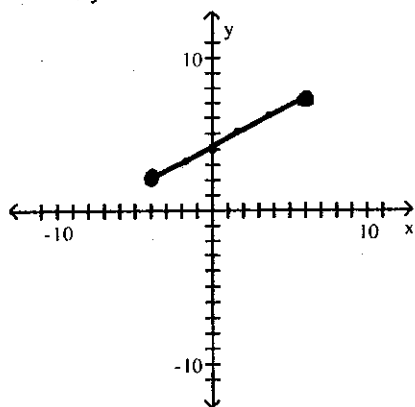
$t = 3$

$x = 2(3) = 6$ $(6, 7)$

$y = 3 + 4 = 7$

3) _____

t	x	y
-2	-4	2
-1	-2	3
0	0	4
1	2	5
2	4	6
3	6	7



Eliminate the parameter.

4) $x = 1 + t, y = t$

2

$t = x - 1$

$y = x - 1$

4) _____

4

5) $x = 2t - 3, y = 9 - 4t, 3 \leq t \leq 5$

$x = 2t - 3 \Rightarrow t = \frac{x+3}{2}$
 $\frac{x+3}{2} = \frac{2t}{2}$

$y = 9 - 4\left(\frac{x+3}{2}\right)$

$y = 9 - 2x - 6$
 $y = -2x + 3$

End points $(3, -3), (7, -11)$

$t = 3$

$x = 2(3) - 3 = 3$

$y = 9 - 4(3) = -3$

$t = 5$

$x = 2(5) - 3 = 7$

$y = 9 - 4(5) = -11$

3 6) $x = t^2, y = t + 1$
 $t = \pm\sqrt{x} \rightarrow Y = \pm\sqrt{x} + 1$
 or
 $Y - 1 = t \rightarrow X = (Y - 1)^2$

6) _____

2 7) $x = t, y = t^3 - 2t + 3$
 $Y = x^3 - 2x + 3$

7) _____

2 8) $x = 4 - t^2, y = t$
 $X = 4 - Y^2$ or $Y = \pm\sqrt{4 - X}$

8) _____

or $x - y = -t^2$
 $\sqrt{x + y} = \sqrt{t^2} \quad t = \pm\sqrt{y - x}$

4 9) $x = t - 3, y = \frac{2}{t}, -5 \leq t \leq 5$
 $x + 3 = t$
 $Y = \frac{2}{x + 3}$

Endpoints: $(-8, \frac{2}{5}) (2, \frac{2}{5})$
 $t = -5 \quad t = 5$
 $x = -5 - 3 = -8 \quad x = 5 - 3 = 2$
 $y = \frac{2}{-5} \quad y = \frac{2}{5}$

9) _____

4 10) $x = 2 \sin t, y = 2 \cos t, 0 \leq t \leq 3\pi/2$
 $x^2 = 4 \sin^2 t$
 $y^2 = 4 \cos^2 t$
 $x^2 + y^2 = 4 \sin^2 t + 4 \cos^2 t$
 $x^2 + y^2 = 4$
 Endpoints: $(0, 2) (-2, 0)$
 $t = 0 \quad t = 3\pi/2$
 $x = 2 \sin 0 = 0 \quad x = 2 \sin 3\pi/2 = -2$
 $y = 2 \cos 0 = 2 \quad y = 2 \cos 3\pi/2 = 0$

10) _____

Solve the problem using a graphing calculator.

4 11) Determine the approximate distance that a baseball travels if it is thrown with a velocity of 97 feet per second at an angle of 30° relative to level ground.
 $x = 97 \cdot \cos 30^\circ t \quad y = -16t^2 + 97 \sin 30^\circ t + 0$
 $x = (97 \cdot \cos 30^\circ)(3.03)$
 $0 = -t(16t - 97 \sin 30^\circ)$
 $-t = 0 \quad t = 16t - 97 \sin 30^\circ$
 $t = 0 \quad t = 3.03$
 $X \approx 255 \text{ ft}$

11) $X = 255 \text{ ft}$

4 12) Estimate the maximum height reached by a baseball during its flight if it is thrown with a velocity of 102 feet per second at an angle of 58° relative to level ground.
 $x = (102 \cos 58^\circ)t \quad y = -16t^2 + (102 \sin 58^\circ)t$
 $t = \frac{-102 \sin 58^\circ}{2(-16)} = 2.7$
 $y(2.7) = -16(2.7)^2 + (102 \sin 58^\circ)(2.7)$
 $= 117 \text{ ft}$

12) 117 ft

4 13) Determine whether a baseball hit 136 feet per second at an angle of 30° relative to level ground will clear a 10-foot wall 400 feet away.

$x = (136 \cos 30^\circ)t \quad y = -16t^2 + (136 \sin 30^\circ)t$
 $400 = (136 \cdot \cos 30^\circ)t$
 $3.396 = t$
 $y(3.396) = -16(3.396)^2 + (136 \sin 30^\circ)(3.396) \approx 46.38 \text{ ft}$
 Yes it will clear the wall.

13) _____

- 4 14) Determine which will travel farther: baseball x hit 102 feet per second at an angle of 35° relative to level ground or baseball y hit 124 feet per second at an angle of 30°. 14) _____

$$x_x = (102 \cos 35) t \quad y_x = -16t^2 + (102 \sin 35) t$$

$$x(3.66) = (102 \cos 35)(3.66) = 305.8 \text{ ft} \quad 0 = -16t^2 + 102 \sin 35 t$$

$$0 = -t(16t - 102 \sin 35) \quad t = 0, t = 3.66 \text{ sec.}$$

$$x_y = (124 \cos 30) t \quad y_y = -16t^2 + (124 \sin 30) t$$

$$x(3.88) = (124 \cos 30)(3.88) = 416.7 \text{ ft.} \quad 0 = -t(16t - 124 \sin 30)$$

$$t = 0 \quad t = 3.88 \text{ sec.}$$

Baseball y

- 4 15) Anne can sprint at a rate of 21 ft/sec. Carol can sprint at 26 ft/sec. Carol gives Anne a 14-ft head start. The parametric equations below can be used to model a race. 15) _____
- $$x_1 = 21t, \quad y_1 = 3$$
- $$x_2 = 26t - 14, \quad y_2 = 5$$

Find a viewing window to simulate a 200-yd dash. Graph simultaneously. Who is ahead after 8 seconds and by how much?

$t = 8$

Anne: $x_1 = 21(8) = 168$

Carol: $x_2 = 26(8) - 14 = 194$

Carol is ahead by 26 feet.

graph:

